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ANALYSIS OF CRACK MOVING AND CURVING IN ANISOTROPIC SOLIDS

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Abstract—The general equations for a dynamically curved crack in an anisotropic solid are derived, and the asymptotic fields of a moving crack under arbitrary distributed loading on the crack surface are calculated from them. For a moving crack under mixed-mode loading conditions a general Muskhelishvili type approach is proposed to calculate intensity factors due to crack surface loading in anisotropic materials. The kinking and curving caused by dynamic loading in anisotropic materials are calculated using the maximum normal stress ratio criterion. The results show that cracks in anisotropic solids may deviate from the straight path and approach a direction parallel to the stiff axis even under symmetric loading and that a crack will tend to deviate more from the crack path to the direction of the stiff axis as the crack speed becomes higher.

1. INTRODUCTION

With the wide applications of composite materials in engineering, fracture in anisotropic materials has been extensively investigated during the past decades. Since crack deviations in anisotropic materials as compared to isotropic materials have different behavior, the study of the crack kinking and curving has received more attention recently (Obata *et al.*, 1989; Gao and Chiu, 1992; Xu and Keer, 1993a).

A general summary of previous research efforts in dynamic fracture has been given by Freund (1990), and also in a series of papers, where he developed important analytical methods for both the steady and non-steady problems of crack propagation. For the anisotropic moving crack problem, Atkinson (1965), and Atkinson and Head (1966) extended the method of Craggs to the moving crack tip analysis. Arcisz and Sih (1984) employed integral transforms to study the local stress field of a moving crack. Wu (1989) used the Stroh (1958) method to derive general expressions of the crack tip moving in an anisotropic material. When a crack tip is moving along a smooth curve, the angular distribution to the first order depends only on the instantaneous velocity of the tip. This result was proved by Freund and Clifton (1974) for isotropic materials, and by Achenbach and Bazant (1975) for anisotropic materials. Viola *et al.* (1989), and Piva and Radi (1991) studied the dynamic response of a steadily moving crack and its time-dependent behavior in an orthotropic media. Non-steady-state asymptotic fields of an accelerating crack were studied by Freund and Rosakis (1992) for isotropic materials, and by Xu and Keer (1993b) for anisotropic materials. In the present paper, the dynamic intensity factors caused by crack surface loading are presented. The calculations indicate that the magnitude of the intensity factors depends only on the distribution of the surface loading, whereas the angular dependence involves the material constants and crack velocity.

In the recent study of crack curving for anisotropic materials, a number of criteria have been proposed for predicting crack deviation, such as the strain energy density ratio criterion (SEDR) by Zhang *et al.* (1990) and damage energy density factor (Z factor) by Zhang *et al.* (1989). The criterion of maximum normal stress ratio (NSR) was proposed by Buczek and Herakovich (1983), and a detailed study of this approach was given by Gregory and Herakovich (1986). In this paper the NSR criterion was used to calculate the direction of crack kinking and curving. The results agree with those given for the static case by Xu and Keer (1993a).

2. FUNDAMENTAL FORMULATION

The elastodynamic equations in an anisotropic solid without body forces are

$$c_{ijkl}\frac{\partial^2 U_k}{\partial X_j\,\partial X_l}=\rho\frac{\partial^2 U_i}{\partial t^2}\quad (i=1,3) \tag{1}$$

where the displacements U_i refer to an original or spatially fixed coordinate system X_i, and c_{ijkl}, ρ are the material constants and density, respectively. Consider a smoothly curved, running crack with local Cartesian coordinates x_i at the crack tip, as shown in Fig. 1, defined so that the origin coincides with the advancing crack tip, and the x_1-axis is oriented towards the direction of crack propagation. The transformation that relates the two systems is given by

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=[\Omega_{ij}]\begin{pmatrix} X_1-a_0(t) \\ X_2-b_0(t) \\ X_3 \end{pmatrix} \tag{2}$$

where $a_0(t)$ and $b_0(t)$ are, respectively, the displacements of the crack tip in the X_1 and Y_1 directions. The matrix $[\Omega_{ij}]$ is

$$[\Omega_{ij}]=\begin{pmatrix} \cos\theta(t) & \sin\theta(t) & 0 \\ -\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3}$$

The displacements in these two coordinate systems are related by

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}=[\Omega_{ij}]\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} \tag{4}$$

where the displacements U_i refer to the original coordinate system, and the displacements u_i to the local or moving coordinate system. Consideration of eqns (2) and (4) yields the governing equation as

$$c^*_{ijkl}\frac{\partial^2 u_k}{\partial x_j\,\partial x_l}=\rho\frac{\mathrm{d}^2 u_i}{\mathrm{d}t^2}+\rho f_i\quad (i=1,2,3) \tag{5}$$

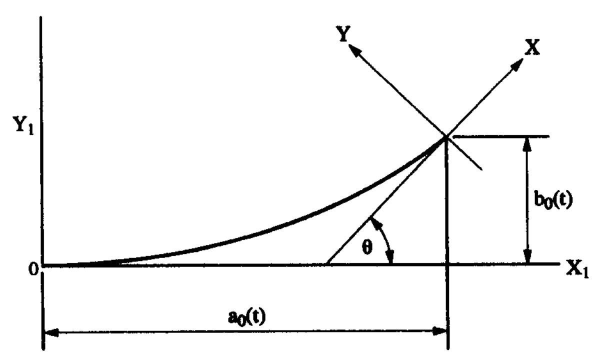

Fig. 1. Coordinate system x–y attached to a running crack tip.

where

$$f_i = 2\frac{\mathrm{d}u_m}{\mathrm{d}t}\dot{\Omega}_{mn}\Omega_{in} + u_m\ddot{\Omega}_{mn}\Omega_{in} \tag{6}$$

and the transformed elastic constants c^*_{ijkl} can be written as $c^*_{ijkl} = \Omega_{ip}\Omega_{jq}\Omega_{kr}\Omega_{ls}c_{pqrs}$. Clearly, $f_3 = 0$, and the functions f_1, f_2 have the form

$$\begin{aligned} f_1(x,y,t) &= -u(x,y,t)\,\dot{\theta}^2 - 2\frac{\mathrm{d}v(x,y,t)}{\mathrm{d}t}\dot{\theta} - v(x,y,t)\ddot{\theta} \\ f_2(x,y,t) &= -v(x,y,t)\,\dot{\theta}^2 + 2\frac{\mathrm{d}u(x,y,t)}{\mathrm{d}t}\dot{\theta} + u(x,y,t)\ddot{\theta} \end{aligned} \tag{7}$$

which are given by substitution of

$$\begin{aligned} \dot{\Omega}_{1n} &= \Omega_{2n}\dot{\theta}, & \dot{\Omega}_{2n} &= -\Omega_{1n}\dot{\theta}, \\ \ddot{\Omega}_{1n} &= -\Omega_{1n}\dot{\theta}^2 + \Omega_{2n}\ddot{\theta}, & \ddot{\Omega}_{2n} &= -\Omega_{2n}\dot{\theta}^2 - \Omega_{1n}\ddot{\theta}, \end{aligned} \tag{8}$$

into eqn (6). Equations (5) combined with (7) are general equations in the moving coordinate system. The functions f_j ($j = 1, 2$) are related to the speed of crack rotation, and the terms f_1 and f_2 vanish for a straight moving crack. When considering the analysis of the higher order asymptotic field for a smoothly curved crack, the functions f_1 and f_2 must be taken into account.

The displacements u_1 and u_2 in the moving coordinate system are written as $u(x, y, t)$ and $v(x, y, t)$. For convenience the constants c^*_{ijkl} will be written as c_{ijkl}, and the formulae are considered in the local coordinate system x–y. Equations (5) can be written as

$$\begin{aligned} a_{11}u_{,xx} + a_{12}u_{,xy} + a_{13}u_{,yy} + a_{21}v_{,xx} + a_{22}v_{,xy} + a_{23}v_{,yy} &= \rho H(u) + \rho G(v) \\ a_{21}u_{,xx} + a_{22}u_{,xy} + a_{23}u_{,yy} + a_{31}v_{,xx} + a_{32}v_{,xy} + a_{33}v_{,yy} &= \rho H(v) - \rho G(u) \end{aligned} \tag{9}$$

where the terms given by operators $H(.)$ and $G(.)$ are related to the crack tip rotation and acceleration. For the first order solutions the terms $H(.)$ and $G(.)$ vanish, for the details of which one can refer to Xu and Keer (1993b).

The coefficients a_{ij} (i, $j = 1, 2, 3$), which depend on the material constants and crack velocity, can be written as

$$\{a_{ij}\} = \begin{pmatrix} C_{11} - \rho v^2 & 2C_{16} & C_{66} \\ C_{16} & C_{12} + C_{66} & C_{26} \\ C_{66} - \rho v^2 & 2C_{26} & C_{22} \end{pmatrix}. \tag{10}$$

The constants C_{mn} of eqn (10) represent the constants c_{ijkl} of eqn (1). An integer function $T_{\text{sub}}(i,j)$, defined for notation purposes only, gives the transformation of subscripts between C_{mn} and C_{ijkl} such that

$$T_{\text{sub}}(i, j) = \tfrac{1}{2}(i+j)\delta_{ij} + (9-i-j)(1-\delta_{ij}) \tag{11}$$

where δ_{ij} is Kronecker's delta. Thus, $m = T_{\text{sub}}(i, j)$ and $n = T_{\text{sub}}(k, l)$.

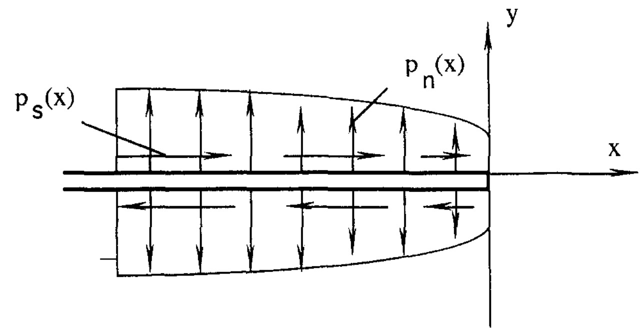

Fig. 2. Moving crack with surface normal loading $p_n(x)$ and shear loading $p_s(x)$.

3. STEADY-STATE SOLUTIONS OF A RUNNING CRACK WITH SURFACE LOADING

In this section an effective means of calculating dynamic intensity factors due to the crack surface loading is proposed for general anisotropic materials. The problem of a semi-infinite crack under arbitrary surface loading will be specifically discussed.

As shown in Fig. 2, normal loading $p_n(x)$ and shear loading $p_s(x)$ are applied to the crack surfaces. The displacement solutions to the homogeneous equations (9) are assumed in the form

$$u = aF(x+\mathrm{i}\beta y), \quad v = bF(x+\mathrm{i}\beta y) \tag{12}$$

where a and b are constants and $F(x+\mathrm{i}\beta y)$ is an analytic function. By substituting eqns (12) into eqns (9), one obtains

$$\begin{pmatrix} Q_{11}(\beta) & Q_{12}(\beta) \\ Q_{12}(\beta) & Q_{22}(\beta) \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \{0\} \tag{13}$$

where the functions $Q_{ij}(\beta)$ are

$$\begin{aligned} Q_{11}(\beta) &= (C_{11}-\rho v^2)+2C_{16}(\mathrm{i}\beta)-C_{66}\beta^2 \\ Q_{12}(\beta) &= C_{16}+(C_{12}+C_{66})(\mathrm{i}\beta)-C_{26}\beta^2 \\ Q_{22}(\beta) &= (C_{66}-\rho v^2)+2C_{26}(\mathrm{i}\beta)-C_{22}\beta^2. \end{aligned} \tag{14}$$

For a non-zero solution to eqns (13) it is required that

$$Q_{11}(\beta)Q_{22}(\beta)-Q_{12}(\beta)Q_{12}(\beta) = 0. \tag{15}$$

If $z_j = x+\mathrm{i}\beta_j y = r_j\,\mathrm{e}^{\mathrm{i}\theta_j}$, $\bar{z}_j = x-\mathrm{i}\bar{\beta}_j y = r_j\,\mathrm{e}^{-\mathrm{i}\theta_j}$, $(j = 1, 2)$, then the displacements to the homogeneous equations, corresponding to eqns (9), are

$$\begin{aligned} u &= \mathrm{Re}\,\{a_1\tilde{Q}(z_1)+a_2\tilde{R}(z_2)\} \\ v &= \mathrm{Re}\,\{b_1\tilde{Q}(z_1)+b_2\tilde{R}(z_2)\} \end{aligned} \tag{16}$$

where (a_j, b_j) the eigenvectors of eqn (13), are

$$\begin{pmatrix} a_j \\ b_j \end{pmatrix} = \begin{pmatrix} -[C_{16}+(C_{12}+C_{66})\mathrm{i}\beta_j-C_{26}\beta_j^2] \\ [(C_{11}-\rho v^2)+2C_{16}\mathrm{i}\beta_j-C_{66}\beta_j^2] \end{pmatrix} \quad (j = 1,2) \tag{17}$$

and β_j $(j = 1, 2)$ are the roots to eqn (15). For isotropic materials $\beta_j = \sqrt{1-(V^2/C_j^2)}$ and are related to the wave velocity. The stresses can be written as

$$\sigma_{xx} = \mathrm{Re}\{M_{11}Q(z_1)+N_{11}R(z_2)\}$$
$$\sigma_{yy} = \mathrm{Re}\{M_{22}Q(z_1)+N_{22}R(z_2)\}$$
$$\sigma_{xy} = \mathrm{Re}\{M_{12}Q(z_1)+N_{12}R(z_2)\}. \tag{18}$$

Here $Q(z) = (\mathrm{d}/\mathrm{d}z)\tilde{Q}(z)$, $R(z) = (\mathrm{d}/\mathrm{d}z)\tilde{R}(z)$ and

$$M_{ij} = (C_{1q}+\mathrm{i}\beta_1 C_{6q})a_1+(C_{6q}+\mathrm{i}\beta_1 C_{2q})b_1$$
$$N_{ij} = (C_{1q}+\mathrm{i}\beta_2 C_{6q})a_2+(C_{6q}+\mathrm{i}\beta_2 C_{2q})b_2 \quad (i,j=1,2) \tag{19}$$

and the subscript $q = T_{\mathrm{sub}}(i, j)$ in (19). The boundary condition on the crack surfaces can be written as

$$-(p_{\mathrm{n}}-\mathrm{i}p_{\mathrm{s}}) = \mathrm{Re}\{M_{22}Q(z_1)+N_{22}R(z_2)\}-\mathrm{i}\,\mathrm{Re}\{M_{12}Q(z_1)+N_{12}R(z_2)\} \tag{20}$$

where the tractions on the crack surfaces are defined as in Fig. 2. The coefficients M_{12}, M_{22}, N_{12} and N_{22} are, in general, complex numbers for anisotropic materials. It can be assumed that

$$Q(z_1) = A_1F(z_1)+B_1G(z_1), \quad R(z_2) = A_2F(z_2)+B_2G(z_2) \tag{21}$$

and $F(t)$, $G(t)$ are real functions. The relations between the coefficients A_j and B_j ($j = 1, 2$) can be determined from the symmetric and antisymmetric properties of tractions p_{n} and p_{s}. It therefore follows that

$$M_{12}A_1+N_{12}A_2 = 0, \quad M_{22}B_1+N_{22}B_2 = 0 \tag{22}$$

whence from eqn (21), the coefficients are

$$A_1 = \frac{N_{12}}{D}, \qquad A_2 = -\frac{M_{12}}{D}$$
$$B_1 = -\frac{N_{22}}{D}, \quad B_2 = \frac{M_{22}}{D} \tag{23}$$

where

$$D = M_{22}N_{12}-M_{12}N_{22}. \tag{24}$$

The roots of function D represent the surface wave speed in the direction of crack propagation. For isotropic materials the root of eqn (24) is the Rayleigh surface wave speed which satisfies the Rayleigh equation, $4\beta_1\beta_2-(1+\beta_2^2)^2 = 0$. The critical velocity of a moving crack in an anisotropic solid should be less than the surface wave speed of the crack propagation direction. For a detailed analysis of free surface wave in anisotropic material, one can refer to the work by Barnett and Lothe (1985). Substitution of eqn (23) into (20) yields

$$-(p_{\mathrm{n}}-\mathrm{i}p_{\mathrm{s}}) = \mathrm{Re}\{F^{\pm}(t)\}-\mathrm{i}\,\mathrm{Re}\{G^{\pm}(t)\}. \tag{25}$$

Using the formulae of Muskhelishvili (1952) and considering the stress distribution at the semi-infinite crack surfaces $(-\infty, h_0)$ gives

$$F(t)-\mathrm{i}G(t) = \frac{1}{\pi\sqrt{t-h_0}}\int_{-\infty}^{h_0}(p_n-\mathrm{i}p_s)\frac{\sqrt{h_0-\xi}}{t-\xi}\,\mathrm{d}\xi \tag{26}$$

where the crack tip is located at $(h_0, 0)$, and h_0 for convenience is taken as 0. The stresses can be written as

$$\sigma_{ij} = \mathrm{Re}\left\{\frac{1}{D}[M_{ij}N_{12}F(z_1)-N_{ij}M_{12}F(z_2)]\right\}$$
$$+\mathrm{Re}\left\{\frac{1}{D}[-M_{ij}N_{22}G(z_1)+N_{ij}M_{22}G(z_2)]\right\} \quad (i,j=1,2) \tag{27}$$

where the coefficients M_{ij} and N_{ij} are given in eqn (19), and

$$F(t) = \frac{1}{\pi\sqrt{t-h_0}}\int_{-\infty}^{h_0}p_n(\xi)\frac{\sqrt{h_0-\xi}}{t-\xi}\,\mathrm{d}\xi,$$
$$G(t) = \frac{1}{\pi\sqrt{t-h_0}}\int_{-\infty}^{h_0}p_s(\xi)\frac{\sqrt{h_0-\xi}}{t-\xi}\,\mathrm{d}\xi, \tag{28}$$

and the displacements can be written as

$$u = \mathrm{Re}\left\{\frac{1}{D}[a_1N_{12}\tilde{F}(z_1)-a_2M_{12}\tilde{F}(z_2)]\right\}+\mathrm{Re}\left\{\frac{1}{D}[-a_1N_{22}\tilde{G}(z_1)+a_2M_{22}\tilde{G}(z_2)]\right\}$$
$$v = \mathrm{Re}\left\{\frac{1}{D}[b_1N_{12}\tilde{F}(z_1)-b_2M_{12}\tilde{F}(z_2)]\right\}+\mathrm{Re}\left\{\frac{1}{D}[-b_1N_{22}\tilde{G}(z_1)+b_2M_{22}\tilde{G}(z_2)]\right\}. \tag{29}$$

Here

$$\tilde{F}(z) = \frac{1}{\pi}\int_{-\infty}^{h_0}p_n(\xi)f(\xi,h_0,z)\,\mathrm{d}\xi, \quad \tilde{G}(z) = \frac{1}{\pi}\int_{-\infty}^{h_0}p_s(\xi)f(\xi,h_0,z)\,\mathrm{d}\xi \tag{30}$$

and

$$f(\xi,h_0,z) = \mathrm{i}\log\left[\frac{\sqrt{h_0-\xi}+\sqrt{h_0-z}}{\sqrt{h_0-\xi}-\sqrt{h_0-z}}\right]. \tag{31}$$

In eqn (27) the first part is related to the mode I loading condition and the second part is related to the mode II loading condition. By considering the asymptotic field at the crack tip, $z_j = h_0+r(\cos\theta+\mathrm{i}\beta_j\sin\theta)$, as $r\to 0$, functions F and G can be written as

$$F(z_j) = \frac{1}{\sqrt{2\pi r}}\sqrt{\frac{2}{\pi}}(\cos\theta+\mathrm{i}\beta_j\sin\theta)^{-1/2}\int_{-\infty}^{h_0}\frac{p_n(\xi)}{\sqrt{h_0-\xi}}\,\mathrm{d}\xi,$$
$$G(z_j) = \frac{1}{\sqrt{2\pi r}}\sqrt{\frac{2}{\pi}}(\cos\theta+\mathrm{i}\beta_j\sin\theta)^{-1/2}\int_{-\infty}^{h_0}\frac{p_s(\xi)}{\sqrt{h_0-\xi}}\,\mathrm{d}\xi. \tag{32}$$

Using the definition of the dynamic stress intensity factors for mode I and II and by substituting eqns (29) into (27), one obtains

$$K_{\rm I} = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{h_0} \frac{p_{\rm n}(\xi)}{\sqrt{h_0-\xi}}\,{\rm d}\xi, \quad K_{\rm II}\sqrt{\frac{2}{\pi}}\int_{-\infty}^{h_0} \frac{p_{\rm s}(\xi)}{\sqrt{h_0-\xi}}\,{\rm d}\xi. \tag{33}$$

The form of the asymptotic field (27) is

$$\sigma_{ij} = \frac{K_{\rm I}}{\sqrt{2\pi r}}\,{\rm Re}\left\{\frac{1}{D}[M_{ij}N_{12}F_1^{-1/2} - N_{ij}M_{12}F_2^{-1/2}]\right\}$$
$$+\frac{K_{\rm II}}{\sqrt{2\pi r}}\,{\rm Re}\left\{\frac{1}{D}[-M_{ij}N_{22}F_1^{-1/2} + N_{ij}M_{22}F_2^{-1/2}]\right\} \quad (i,j=1,2) \tag{34a}$$

where

$$F_j = \cos\theta + {\rm i}\beta_j \sin\theta, \quad (j=1,2) \tag{34b}$$

and M_{ij}, N_{ij} are constants determined by the material property. In terms of eqn (30) and (31) the asymptotic form of functions $\tilde{F}(z_j)$ and $\tilde{G}(z_j)$ can be

$$\tilde{F}(z_j) = \sqrt{\frac{r}{2\pi}}\,2\sqrt{\frac{2}{\pi}}(\cos\theta + {\rm i}\beta_j\sin\theta)^{1/2}\int_{-\infty}^{h_0}\frac{p_{\rm n}(\xi)}{\sqrt{h_0-\xi}}\,{\rm d}\xi, \tag{35}$$
$$\tilde{G}(z_j) = \sqrt{\frac{r}{2\pi}}\,2\sqrt{\frac{2}{\pi}}(\cos\theta + {\rm i}\beta_j\sin\theta)^{1/2}\int_{-\infty}^{h_0}\frac{p_{\rm s}(\xi)}{\sqrt{h_0-\xi}}\,{\rm d}\xi,$$

therefore the displacement can be written as

$$u = 2\sqrt{\frac{r}{2\pi}}K_{\rm I}\,{\rm Re}\left\{\frac{1}{D}[a_1N_{12}F_1^{1/2} - a_2M_{12}F_2^{1/2}]\right\}$$
$$+2\sqrt{\frac{r}{2\pi}}K_{\rm II}\,{\rm Re}\left\{\frac{1}{D}[-a_1N_{22}F_1^{1/2} + a_2M_{22}F_2^{1/2}]\right\}$$

$$v = 2\sqrt{\frac{r}{2\pi}}K_{\rm I}\,{\rm Re}\left\{\frac{1}{D}[b_1N_{12}F_1^{1/2} - b_2M_{12}F_2^{1/2}]\right\}$$
$$+2\sqrt{\frac{r}{2\pi}}K_{\rm II}\,{\rm Re}\left\{\frac{1}{D}[-b_1N_{22}F_1^{1/2} + b_2M_{22}F_2^{1/2}]\right\}. \tag{36}$$

The hoop stress $\sigma_{\theta\theta}$ at the crack tip can be written as $\sigma_{\theta\theta} = (K_I/\sqrt{2\pi r})\,f_{\theta\theta}(\theta)$, where the function $f_{\theta\theta}$ depends on the crack velocity V, the material constants, such as the degree of anisotropy L ($L = E_1/E_2$), as well as the crack orientation ϕ; here ϕ is the angle made between the crack axis and the stiff axis of an anisotropic material. Figures 3a, b and c show the angular variation of the hoop stress for various velocities and crack orientations. Functions $f_{\theta\theta}(\theta)$ for a moving crack with velocity $V = 0.15\sqrt{E_2/\rho}$ in the stiff axis direction for three values of L (L = 1, 3, 8) are shown in Fig. 3a. The similar cases for velocity $V = 0.3\sqrt{E_2/\rho}$ are shown in Fig. 3b. It can be seen that the maximum value will be higher as the degree of anisotropy increases, which implies that the crack branching tendency will be higher as the degree of anisotropy is higher. However, when the crack propagation is at an angle of 30° with the stiff axis as shown in Fig. 3, the crack branching tendency will be low as the degree of anisotropy increases.

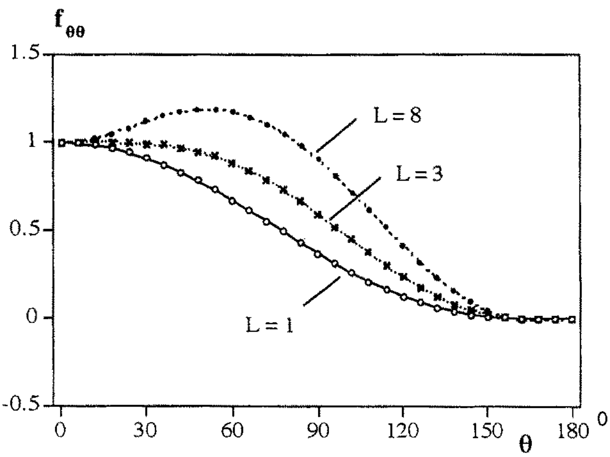

Fig. 3a. Angular variation of $\sigma_{\theta\theta}$ ($V = 0.15\sqrt{E_1/\rho}$) for crack propagation in the stiff axis direction.

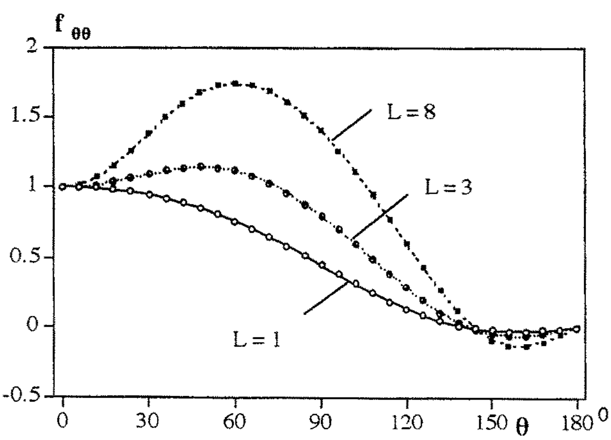

Fig. 3b. Angular variation of $\sigma_{\theta\theta}$ ($V = 0.3\sqrt{E_1/\rho}$) for crack propagation in the stiff axis direction.

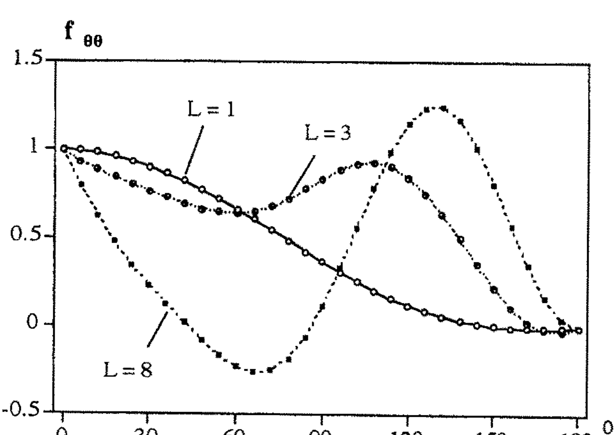

Fig. 3c. Angular variation of $\sigma_{\theta\theta}$ ($V = 0.15\sqrt{E_1/\rho}$) for crack propagation in the direction making a 30° angle with the stiff axis.

The specific case of a concentrated point load at $(-L, 0)$ is considered. For this case the functions in eqn (28) can be written as

$$F(z_j) = \frac{p_n}{\pi\sqrt{z_j-h_0}}\frac{\sqrt{h_0+L}}{z_j+L}, \quad G(z_j) = \frac{p_s}{\pi\sqrt{z_j-h_0}}\frac{\sqrt{h_0+L}}{z_j+L} \quad (j = 1,2) \tag{37}$$

where p_n and p_s are the strengths of the concentrated load acting in the normal and tangential directions. Therefore, eqn (32) becomes

$$F(z_j) = \frac{1}{\sqrt{2\pi r}}\sqrt{\frac{2}{\pi L}}\,p_n(\cos\theta+\mathrm{i}\beta_j\sin\theta)^{-1/2}$$

$$G(z_j) = \frac{1}{\sqrt{2\pi r}}\sqrt{\frac{2}{\pi L}}\,p_s(\cos\theta+\mathrm{i}\beta_j\sin\theta)^{-1/2}. \tag{38}$$

Substituting eqn (33) into (27) yields

$$\sigma_{ij} = \frac{1}{\sqrt{2\pi r}}\sqrt{\frac{2}{\pi L}}\,p_n\mathrm{Re}\left\{\frac{1}{D}[M_{ij}N_{12}F_1^{-1/2}-N_{ij}M_{12}F_2^{-1/2}]\right\}$$
$$+\frac{1}{\sqrt{2\pi r}}\sqrt{\frac{2}{\pi L}}\,p_s\mathrm{Re}\left\{\frac{1}{D}[-M_{ij}N_{22}F_1^{-1/2}+N_{ij}M_{22}F_2^{-1/2}]\right\} \tag{39}$$

where the coefficients M_{ij}, N_{ij} are listed in eqn (19), and functions F_1, F_2 are listed in (34b). From eqn (39) it can be seen that the intensity factors for modes I and II are

$$K_{\mathrm{I}} = \sqrt{\frac{2}{\pi L}}\,p_n, \quad K_{\mathrm{II}} = \sqrt{\frac{2}{\pi L}}\,p_s,$$

which are identical with those for the static problem. The energy release rate G can be written as

$$G = \tfrac{1}{2}\{G_{11}K_{\mathrm{I}}^2+G_{12}K_{\mathrm{I}}K_{\mathrm{II}}+G_{22}K_{\mathrm{II}}^2\} \tag{40}$$

where

$$G_{11} = -\mathrm{Im}\left\{\frac{1}{D}(b_1N_{12}-b_2M_{12})\right\}$$

$$G_{12} = \mathrm{Im}\left\{\frac{1}{D}[(b_1N_{22}-b_2M_{22})-(a_1N_{12}-a_2M_{12})]\right\}$$

$$G_{22} = \mathrm{Im}\left\{\frac{1}{D}(a_1N_{22}-a_2M_{22})\right\}. \tag{41}$$

For moving cracks in isotropic materials eqn (40) becomes

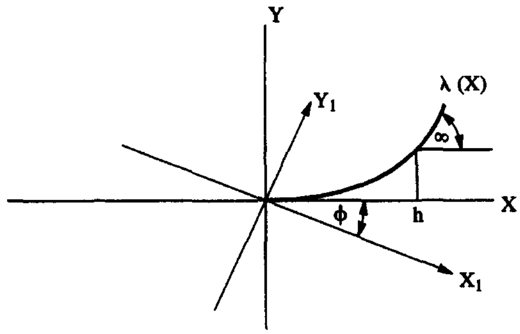

Fig. 4. Crack with curved extension in anisotropic solids.

$$G = \frac{\beta_1}{4\beta_1\beta_2-(1+\beta_2^2)^2}\frac{V^2}{2\mu C_2}K_{\mathrm{I}}^2 + \frac{\beta_2}{4\beta_1\beta_2-(1+\beta_2^2)^2}\frac{V^2}{2\mu C_2}K_{\mathrm{II}}^2 \tag{42}$$

as given by Freund (1990). For the steady-state moving crack with a surface concentrated loading p applied at the point $(-L, 0)$, the energy release rate G can be written as

$$G = \frac{\beta_1}{4\beta_1\beta_2-(1+\beta_2^2)^2}\frac{V^2}{2\mu C_2}K_{\mathrm{IC}}^2 = \frac{\beta_1}{4\beta_1\beta_2-(1+\beta_2^2)^2}\frac{V^2}{\mu C_2}\frac{p^2}{\pi L}. \tag{43}$$

Thus

$$p = K_{\mathrm{IC}}\sqrt{\frac{\pi L}{2}}. \tag{44}$$

For the problem of a moving crack with length $2a$ and surface loading P_n and P_s, the functions $F(t)$ and $G(t)$, given in terms of eqn (25), are

$$F(t) = \frac{1}{\pi\sqrt{t^2-a^2}}\int_{-a}^{a} P_n(\xi)\frac{\sqrt{a^2-\xi^2}}{t-\xi}\,d\xi, \tag{45}$$

$$G(t) = \frac{1}{\pi\sqrt{t^2-a^2}}\int_{-a}^{a} P_s(\xi)\frac{\sqrt{a^2-\xi^2}}{t-\xi}\,d\xi.$$

The intensity factor can be derived from eqn (34) as

$$K_{\mathrm{I}} = \frac{1}{\sqrt{\pi a}}\int_{-a}^{a} p_n(\xi)\sqrt{\frac{a+\xi}{a-\xi}}\,d\xi$$

$$K_{\mathrm{II}} = \frac{1}{\sqrt{\pi a}}\int_{-a}^{a} p_s(\xi)\sqrt{\frac{a+\xi}{a-\xi}}\,d\xi. \tag{46}$$

The expression of intensity factor (33) or (46) for a moving crack has the same form as that of the static crack. Solutions for the moving crack of constant length may be obtained by an analogous analysis.

4. STRESS ANALYSIS AT THE MOVING CRACK TIP

The slightly deviated shape of the semi-infinite crack extension (see Fig. 4) can be approximated as

$$\lambda(x) = \alpha_1 x + \alpha_2 x^{3/2} + O(x^2). \tag{47}$$

This expression (47) has been used by Cotterell and Rice (1980) for the isotropic problem, where the shape parameters of the crack extension are α_1, α_2 and the deviation from the crack path is assumed to be small. The crack is also assumed to be opened by normal and shear tractions, T_n and T_s. Using eqns (18) gives the stress as

$$\begin{aligned} \sigma_{xx}+\sigma_{yy} &= \mathrm{Re}\,\{(M_{11}+M_{22})\,Q(z_1)+(N_{11}+N_{22})R(z_2)\} \\ \sigma_{yy}-\sigma_{xx}-2\mathrm{i}\,\sigma_{xy} &= \mathrm{Re}\,\{(M_{22}-M_{11})Q(z_1)+(N_{22}-N_{11})R(z_2)\} \\ &\quad -2\mathrm{i}\,\mathrm{Re}\,\{M_{12}Q(z_1)+N_{12}R(z_2)\} \end{aligned} \tag{48}$$

and the tractions as

$$-(T_n - \mathrm{i}T_s) = \tfrac{1}{2}\{(\sigma_{xx}+\sigma_{yy})+\mathrm{e}^{-2\mathrm{i}\xi}(\sigma_{yy}-\sigma_{xx}-2\mathrm{i}\sigma_{xy})\} \tag{49}$$

or

$$\begin{aligned} -(T_n-\mathrm{i}T_s) &= \tfrac{1}{2}\{\mathrm{Re}\,[(M_{11}+M_{22})Q(z_1)+(N_{11}+N_{22})R(z_2)]\} \\ &\quad + \mathrm{e}^{-2\mathrm{i}\xi}\{\mathrm{Re}\,[(M_{22}-M_{11})Q(z_1)+(N_{22}-N_{11})R(z_2)] \\ &\quad -2\mathrm{i}\,\mathrm{Re}\,[M_{12}Q(z_1)+N_{12}R(z_2)]\}. \end{aligned} \tag{50}$$

Here ξ refers to the angle made by the crack relative to the x-axis. By expanding eqns (48)–(50) into powers of λ, ($\lambda \ll 1$), where λ represents the deviation of the actual crack from the straight path, the functions $Q(z_1)$ and $R(z_2)$ can be expressed as

$$Q(z_1) = Q_0(z_1)+Q_1(z_1)+O(\lambda^2), \quad R(z_2) = R_0(z_2)+R_1(z_2)+O(\lambda^2). \tag{51}$$

Here, $Q_0(z_1)$ and $R_0(z_2)$ are given terms of $O(\lambda^0)$, and $Q_1(z_1)$ and $R_1(z_2)$ are of $O(\lambda^1)$, as $\lambda \to 0$. Thus the boundary conditions on the crack $Q_{,z1}$ and $R_{,z2}$ at the position $z = t+\mathrm{i}\lambda(t)$ are given by

$$Q^{\pm}(z_1) = Q_0^{\pm}(t)+\mathrm{i}\beta_1\lambda\left(\frac{\mathrm{d}Q_0}{\mathrm{d}z_1}\right)_t^{\pm}+Q_1^{\pm}(t) \tag{52}$$

$$R^{\pm}(z_2) = R_0^{\pm}(t)+\mathrm{i}\beta_2\lambda\left(\frac{\mathrm{d}R_0}{\mathrm{d}z_2}\right)_t^{\pm}+R_1^{\pm}(t). \tag{53}$$

By noting that $\xi = \lambda'(t)$, the boundary conditions on the crack, at $z = t+\mathrm{i}\lambda(t)$, become

$$\begin{aligned} -(T_n-\mathrm{i}T_s) &= \mathrm{Re}\,[M_{22}Q_0+N_{22}R_0]-\mathrm{i}\,\mathrm{Re}\,[M_{12}Q_0+N_{12}R_0] \\ &\quad + \mathrm{Re}\,[M_{22}Q_1+N_{22}R_1]-\mathrm{i}\,\mathrm{Re}\,[M_{12}Q_1+N_{12}R_1] \\ &\quad +\lambda\,\{\mathrm{Re}\,[\mathrm{i}\beta_1 M_{22}Q_0'+\mathrm{i}\beta_2 N_{22}R_0']-\mathrm{i}\,\mathrm{Re}\,[\mathrm{i}\beta_1 M_{12}Q_0'+\mathrm{i}\beta_2 N_{12}R_0']\} \\ &\quad -\lambda'\{2\,\mathrm{Re}\,[M_{12}Q_0+N_{12}R_0]+\mathrm{i}\,\mathrm{Re}\,[(M_{22}-M_{11})Q_0+(N_{22}-N_{11})R_0]\}. \end{aligned} \tag{54}$$

For the static problem, eqn (54) becomes identical to the result of Xu and Keer (1993a). Functions $Q(z_1)$ and $R(z_2)$ are holomorphic in the whole plane cut along the crack and may be written as

$$Q(z_1) = [N_{12}F(z_1) - N_{22}G(z_1)]/D$$
$$R(z_2) = [-M_{12}F(z_2) + M_{22}G(z_2)]/D. \tag{55}$$

Here, the functions $F(z)$ and $G(z)$ have the same behavior as $Q(z_1)$ and $R(z_2)$, and $F(t)$ and $G(t)$ are real functions. Therefore eqn (54) can be written as

$$\begin{aligned} -(T_n - iT_s) = &\operatorname{Re}[F_0(t)] - i\operatorname{Re}[G_0(t)] + \operatorname{Re}[F_1(t)] - i\operatorname{Re}[G_1(t)] \\ &-\lambda\{\operatorname{Im}[m_1F_0'(t) + m_2G_0'(t)] + i\operatorname{Im}[m_3F_0'(t)] + m_4G_0'(t)]\} \\ &-\lambda'\{\operatorname{Re}[n_1F_0(t) + n_2G_0(t)] + i\operatorname{Re}[n_3F_0(t) + n_4G_0(t)]\}, \end{aligned} \tag{56}$$

where the coefficients m_j, n_j ($j = 1, 2, 3, 4$) are listed in Appendix A.

By separating the terms of $O(\lambda^0)$ and $O(\lambda^1)$ in eqn (56), using the formulae of Muskhelishvili (1952), and considering only the redistribution of stresses caused by crack extension ($T_n = T_s = 0$, at $-\infty \leqslant t \leqslant 0$),

$$F_0(t) - iG_0(t) = \frac{1}{\pi\sqrt{t-h}} \int_0^h (T_n - iT_s) \frac{\sqrt{h-\xi}}{t-\xi} d\xi \tag{57}$$

$$\begin{aligned} F_1(t) - iG_1(t) = \frac{1}{\pi\sqrt{t-h}} \int_0^h &\{\lambda[\operatorname{Im}(m_1)T_n' + \operatorname{Im}(m_2)T_s' + i\operatorname{Im}(m_3)T_n' + i\operatorname{Im}(m_4)T_s'] \\ &+\lambda'[\operatorname{Re}(n_1)T_n + \operatorname{Re}(n_2)T_s + i\operatorname{Re}(n_3)T_n + i\operatorname{Re}(n_4)T_s]\} \frac{\sqrt{h-\xi}}{t-\xi} d\xi \end{aligned} \tag{58}$$

where T_n and T_s are defined in terms of the shape of the crack extension and the stress distribution ahead of the original crack tip. These tractions can be written as

$$T_n = \sigma_{22}^0 - 2\lambda'\sigma_{12}^0 + \lambda\sigma_{22,2}^0$$
$$T_s = \lambda'(\sigma_{22}^0 - \sigma_{11}^0) + \sigma_{12}^0 + \lambda\sigma_{12,2}^0. \tag{59}$$

Here, σ_{ij}^0 represents $\sigma_{ij}(x, y)|_{y=0}$ and the stress σ_{ij}^0 and its derivative $\sigma_{ij,2}^0$ are given as follows:

$$\sigma_{11}^0 = \frac{1}{\sqrt{2\pi x}}\{(1-t_1)K_I - t_2K_{II}\} + T$$

$$\sigma_{22}^0 = \frac{1}{\sqrt{2\pi x}}K_I, \qquad \sigma_{12}^0 = \frac{1}{\sqrt{2\pi x}}K_{II}$$

$$\sigma_{22,2}^0 = \frac{1}{2x\sqrt{2\pi x}}\{s_1K_I + s_2K_{II}\}, \quad \sigma_{12,2}^0 = \frac{1}{2x\sqrt{2\pi x}}\{s_3K_I + s_4K_{II}\}. \tag{60}$$

The coefficients t_j ($j = 1, 2$) and s_j ($j = 1, 2, 3, 4$) are listed in Appendix A. Equations (59) can be expressed as

$$T_n = \frac{1}{\sqrt{2\pi x}}[k_I(1 + \tfrac{1}{2}s_1\alpha_1) + \alpha_1k_{II}(\tfrac{1}{2}s_2 - 2)] + \frac{1}{\sqrt{2\pi}}\alpha_2[\tfrac{1}{2}s_1k_I + (\tfrac{1}{2}s_2 - 3)k_{II}]\sqrt{h} \tag{61}$$

$$\begin{aligned} T_s = &\frac{1}{\sqrt{2\pi x}}\{k_{II}[1 + \alpha_1(t_2 + \tfrac{1}{2}s_4)] + \alpha_1k_I(t_1 + \tfrac{1}{2}s_3)\} \\ &+ \frac{1}{\sqrt{2\pi}}\{\tfrac{1}{2}\alpha_2k_I(3t_1 + s_3) + \tfrac{1}{2}\alpha_2k_{II}(3t_2 + s_4) - \sqrt{2\pi}\,\alpha_1T\}\sqrt{h}. \end{aligned} \tag{62}$$

Here $k_{\rm I}$, $k_{\rm II}$ denote the stress intensity factors at original crack tip, and the T stress is denoted by T. For the static problem, the expressions for the tractions in eqns (61) and (62) reduce to the leading terms derived by Xu and Keer (1993a).

By letting ω be the slope of the crack tip at $x = h$ (Fig. 4), the normal and shear stresses, $\sigma_{\omega\omega}$ and $\sigma_{r\omega}$, acting at a small distance r from the tip across its extension, are obtained by substitution of $F_0(z)$, $F_1(z)$ and $G_0(z)$, $G_1(z)$ in eqn (15) and by identification of terms in $-(T_{\rm n}-{\rm i}T_{\rm s})$ with $\sigma_{\omega\omega}-{\rm i}\sigma_{r\omega}$. Thus,

$$\begin{aligned}\sigma_{\omega\omega}-{\rm i}\sigma_{r\omega} &= F_0(h+r)-{\rm i}G_0(h+r)+F_1(h+r)-{\rm i}G_1(h+r)\\ &\quad -\omega r\{[{\rm Im}\,(m_1)+{\rm i}\,{\rm Im}\,(m_3)]F_0'(h+r)+[{\rm Im}\,(m_2)+{\rm i}\,{\rm Im}\,(m_4)]G_0'(h+r)\}\\ &\quad -\omega\{[{\rm Re}\,(n_1)+{\rm i}\,{\rm Re}\,(n_3)]F_0(h+r)+[{\rm Re}\,(n_2)+{\rm i}\,{\rm Re}\,(n_4)]G_0(h+r)\}. \qquad (63)\end{aligned}$$

By substitution of eqns (57) and (58) into (63) and letting $r \to 0$, it follows that

$$\sigma_{\omega\omega}-{\rm i}\sigma_{r\omega} = \frac{1}{\pi\sqrt{r}}\int_0^h (q_{\rm I}-{\rm i}q_{\rm II})\frac{{\rm d}t}{\sqrt{h-t}} \qquad (64)$$

where

$$\begin{aligned}q_{\rm I} &= T_{\rm n}+[\tfrac{1}{2}{\rm Im}\,(m_1)-{\rm Re}\,(n_1)]\omega T_{\rm n}+[\tfrac{1}{2}{\rm Im}\,(m_2)-{\rm Re}\,(n_2)]\omega T_{\rm s}+{\rm Im}\,(m_1)\lambda T_{\rm n}'\\ &\quad +{\rm Im}\,(m_2)\lambda T_{\rm s}'+{\rm Re}\,(n_1)\lambda' T_{\rm n}+{\rm Re}\,(n_2)\lambda' T_s\\ q_{\rm II} &= T_{\rm s}-[\tfrac{1}{2}{\rm Im}\,(m_3)-{\rm Re}\,(n_3)]\omega T_{\rm n}-[\tfrac{1}{2}{\rm Im}\,(m_4)-{\rm Re}\,(n_4)]\omega T_{\rm s}-{\rm Im}\,(m_3)\,\lambda T_{\rm n}'\\ &\quad -{\rm Im}\,(m_4)\lambda T_{\rm s}'-{\rm Re}\,(n_3)\lambda' T_{\rm n}-{\rm Re}\,(n_4)\lambda' T_{\rm s}. \qquad (65)\end{aligned}$$

For the anisotropic static problem, $q_{\rm I}$ and $q_{\rm II}$ are identical to Xu and Keer (1993a), and for the isotropic static problem they are identical to Cotterell and Rice (1980).

By substituting eqns (27), (28) into (30) and noting that

$$\sigma_{\omega\omega}-{\rm i}\sigma_{r\omega} = \frac{1}{\sqrt{2\pi r}}(K_{\rm I}-{\rm i}K_{\rm II}), \qquad (66)$$

the crack tip intensity factors are given as

$$\begin{aligned}K_{\rm I} &= [k_{\rm I}+p_1\alpha_1 k_{\rm I}+p_2\alpha_1 k_{\rm II}]+[p_3\alpha_2 k_{\rm I}+p_4\alpha_2 k_{\rm II}]\sqrt{h}\\ K_{\rm II} &= [k_{\rm II}+q_1\alpha_1 k_{\rm I}+q_2\alpha_1 k_{\rm II}]+\left[q_3\alpha_2 k_{\rm I}+q_4\alpha_2 k_{\rm II}-2\sqrt{\frac{2}{\pi}}\alpha_1 T\right]\sqrt{h}. \qquad (67{\rm a,b})\end{aligned}$$

Here p_j and q_j ($j = 1, 2, 3, 4$) are listed in Appendix B. For the anisotropic static problem, eqns (67a) become

$$\begin{aligned}K_{\rm I} &= (k_{\rm I}-\tfrac{3}{2}\alpha_1 k_{\rm II})-\tfrac{9}{4}\alpha_2 k_{\rm II}\sqrt{h}\\ K_{\rm II} &= \{k_{\rm II}[1-\tfrac{1}{2}\alpha_1\,{\rm Im}\,(\beta_1+\beta_2)]+\tfrac{1}{2}\alpha_1 k_{\rm I}\,{\rm Re}\,(2-\beta_1\beta_2)\}\\ &\quad +\left\{\tfrac{3}{4}\alpha_2 k_{\rm I}\,{\rm Re}\,(2-\beta_1\beta_2)-\tfrac{3}{4}\alpha_2 k_{\rm II}\,{\rm Im}\,(\beta_1+\beta_2)-2\sqrt{\frac{2}{\pi}}\beta_1 T\right\}\sqrt{h}. \qquad (68{\rm a,b})\end{aligned}$$

Equations (68a, b) agree with the corresponding equations by Xu and Keer (1993a).

5. ANALYSIS OF CRACK KINKING AND CURVING

The NSR criterion will be used to calculate the crack deviation, since compared to the other criteria, it is mathematically more convenient. For isotropic materials, the direction of the maximum normal stress is essentially the direction of $K_{II} = 0$. The direction of crack kinking in anisotropic materials may not always occur along the direction of $K_{II} = 0$ or in the direction of the maximum hoop stress. Therefore the ratio $\sigma_{\theta\theta}/T_{\theta\theta}$ may be crucial for predicting the crack growth in such materials. The detailed theoretical analysis using the NSR criterion is given by Xu and Keer (1993a), and the form of the intensity factors at the crack tip is given as

$$K_{II} = f(\phi)K_I, \quad f(\phi) = -\frac{2T'_{\theta\theta}(\phi)}{3T_{\theta\theta}(\phi)} \tag{69a,b}$$

where $T_{\theta\theta}(\phi)$, which represents the tensile strength in the direction perpendicular to the crack orientation ϕ (see Fig. 5), can be written as $T_{\theta\theta}(\phi) = X_T \sin^2\phi + Y_T \cos^2\phi$. The relatively simple form of eqn (69) can be written as

$$K_{II} = -\frac{2(X_T - Y_T)\sin 2\phi}{3(X_T \sin^2\phi + Y_T \cos^2\phi)} K_I \tag{70}$$

where X_T and Y_T denote, respectively, the tensile strength along the stiffer and more compliant axes. For an isotropic material, the crack will always kink approximately towards the direction of $K_{II} = 0$ since $X_T = Y_T$ in eqn (70).

Substituting eqns (67a, b) into (70) and considering crack orientation $\phi + \alpha$ after kinking, the following results for the crack kinking and curving are obtained:

$$\alpha_1 = \frac{f(\phi)K_I - K_{II}}{K_I[q_1 - f(\phi)p_1 - f'(\phi)] + K_{II}[q_2 - f(\phi)p_2]} \tag{71}$$

and

$$\alpha_2 = \frac{2\sqrt{\frac{2}{\pi}}\alpha_1 T}{K_I[q_3 - f(\phi)p_3] + K_{II}[q_4 - f(\phi)p_4] - \frac{3}{2}K_I f'(\phi)}. \tag{72}$$

For the static anisotropic problem eqns (71) and (72) reduce to the result of Xu and Keer

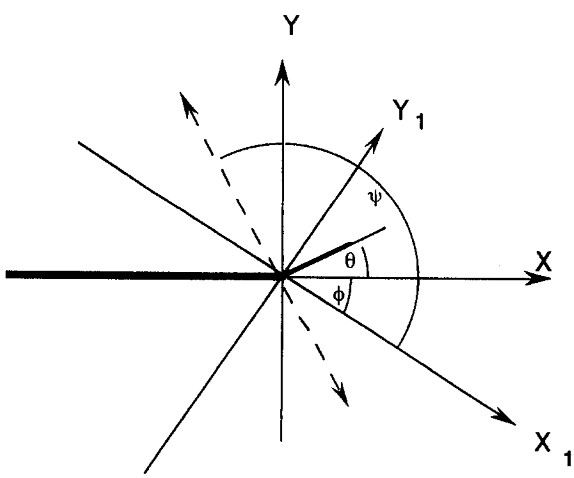

Fig. 5. The tensile strength T_s is the function of ψ ($\psi = \phi + \theta + \pi/2$).

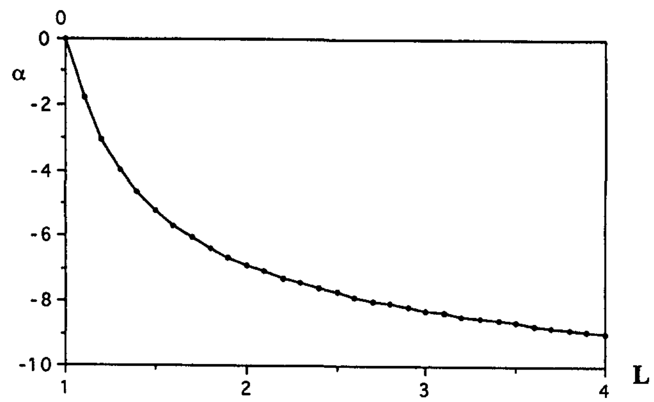

Fig. 6. Crack deviation vs degree of anisotropy.

(1993a). For the mode I loading condition, $K_{II} = 0$, and the crack deviation is a function of the material constants and crack velocity. Figure 6 shows that the kink angle is a function of the degree of anisotropy for a crack moving in the direction an angle of 8° relative to the stiff axis and at the speed of $V = \sqrt{0.1E_1/\rho}$. As the degree of anisotropy L ($L = E_1/E_2 = X_T/Y_T$) increases, the angle of crack deviation increases towards the direction parallel to the stiff axis. Figure 7 shows that crack kinking occurs under various crack velocities. As the crack velocity ($V = k\sqrt{E_1/\rho}$) increases, the crack will deviate more from the crack path to the direction parallel to the stiff axis. For the special case of isotropic materials, eqns (71) and (72) are

$$\alpha_1 = -2\frac{K_{II}}{K_I}, \quad \alpha_2 = \frac{8}{3}\sqrt{\frac{2}{\pi}}\frac{\alpha_1 T}{K_I} \tag{73}$$

given by Sumi *et al.* (1983).

The crack path stability in isotropic materials has been studied by Cotterell and Rice (1980). The crack path is stable when the T stress is negative and is unstable when it is positive. Xu and Keer (1993a) studied the crack path stability in anisotropic materials and concluded that the stability of the crack depends on the material constants as well as the T stress. For the dynamic problem, the crack path stability also depends on the crack velocity. The analysis is based on the maximum normal stress ratio criterion with the crack tip condition of eqn (69), $K_{II} = f(\phi)K_I$. Moreover, other criteria can be used to analyse the crack deviation and forms similar to that of eqn (69) will result.

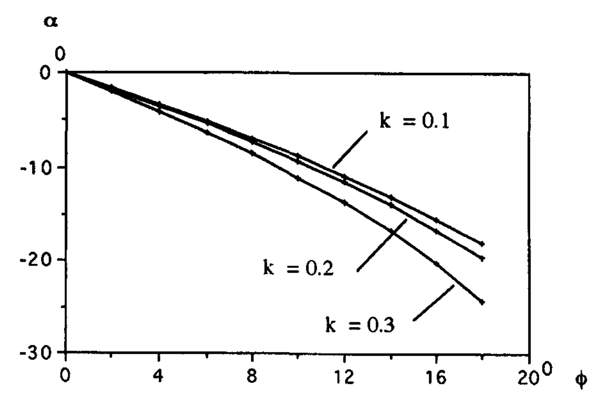

Fig. 7. Curves showing kink angle increase with increasing velocity.

6. CONCLUSION

The asymptotic field for a moving crack with surface loading can be calculated using the moving coordinate system and Muskhelishvili approach. The results show that the intensity factors of a semi-infinite moving crack depend only on the distribution of the surface loading whether for anisotropic materials or isotropic materials. This result can be well used to simplify the calculation of intensity factors for anisotropic materials. The angular distribution functions of the asymptotic fields depend on the crack velocity and material properties.

In terms of the maximum normal stress ratio criterion, a running crack will move in the direction of $K_{\rm II} = f(\phi)K_{\rm I}$. The function $f(\phi)$ is zero only when either the crack tip is parallel to a principal axis or the tensile strengths are isotropic. Under symmetric loading conditions the crack may deviate from the straight path. The calculations show that there is crack deviation from the crack path as the crack velocity is larger.

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APPENDIX A

The coefficients m_j, n_j in eqn (56) are as follows:

$$m_1 = (\beta_1 M_{22}N_{12} - \beta_2 N_{22}M_{12})/D, \qquad m_2 = (-\beta_1 M_{22}N_{22} + \beta_2 N_{22}M_{22})/D,$$

$$m_3 = (-\beta_1 M_{12}N_{12} + \beta_2 N_{12}M_{12})/D, \qquad m_4 = (\beta_1 M_{12}N_{22} - \beta_2 N_{12}M_{22})/D,$$

$n_1 = 0,$ $\quad n_2 = 2,$

$n_3 = [(M_{22}-M_{11})N_{12}-(N_{22}-N_{11})M_{12}]/D,$ $\quad n_4 = [-(M_{22}-+M_{11})N_{22}+(N_{22}-N_{11})M_{22}]/D,$

$t_1 = 1-\mathrm{Re}\left[\frac{1}{D}(M_{11}N_{12}-N_{11}M_{12})\right],$ $\quad t_2 = \mathrm{Re}\left[\frac{1}{D}(M_{11}N_{22}-N_{11}M_{22})\right],$

$s_1 = \mathrm{Im}\,(m_1),$ $\quad s_2 = \mathrm{Im}\,(m_2),$

$s_3 = -\mathrm{Im}\,(m_3),$ $\quad s_4 = -\mathrm{Im}\,(m_4),$

$D = M_{22}N_{12}-N_{22}M_{12}.$

APPENDIX B

The coefficients p_j, q_j $(j = 1, 2, 3, 4)$ are as follows:

$p_1 = \frac{1}{2}\mathrm{Im}\,(m_1),$ $\quad p_2 = \frac{1}{2}\mathrm{Im}\,(m_2)-2,$

$p_3 = \frac{3}{4}\mathrm{Im}\,(m_1),$ $\quad p_4 = \frac{3}{4}\mathrm{Im}\,(m_3)-3,$

$q_1 = 1-\frac{1}{2}\mathrm{Im}\,(m_3)-\mathrm{Re}\,(X),$ $\quad q_2 = \mathrm{Re}\,(n_4)-\frac{1}{2}\mathrm{Im}\,(m_4),$

$$q_3 = \tfrac{3}{4}[2\mathrm{Re}\,(n_3)-\mathrm{Im}\,(m_3)]+\frac{3}{\pi}[1-\mathrm{Re}\,(X)-\mathrm{Re}\,(n_3)],$$

$$q_4 = \tfrac{3}{4}[2Re\,(n_4)-\mathrm{Im}\,(m_4)],$$

$$X = \frac{1}{D}(M_{11}N_{12}-N_{11}M_{12}).$$